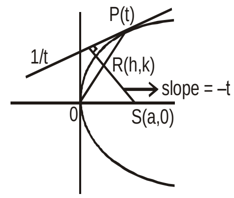
EXERCISE – II
HINTS & SOLUTIONS

Sol.1 **B**

Equation of OP



$y = \frac{2}{t}x$

$k = \frac{2}{t}h$ ....(1)

$y - 0 = -t(x - a) \Rightarrow y = -tx + at$

$\Rightarrow k = -th + at \Rightarrow \frac{2}{t}h = -th + at$ from (1)

$(t = \frac{2h}{k})$

$$h = \frac{at^2}{2+t^2} \Rightarrow h = \frac{a\frac{4h^2}{k^2}}{2+\frac{4h^2}{k^2}} \Rightarrow h = \frac{2ah^2}{k^2+2h^2}$$

$\Rightarrow k^2 + 2h^2 = 2ah \Rightarrow 2x^2 + y^2 - 2ax = 0$

Sol.2 **A,B**

$y^2 - 2y - 4x - 7 = 0$

$y^2 - 2y + 1 - 4x - 8 = 0$ LR = 4 = L

$(y - 1)^2 = 4(x + 2)$

vertex $(-2, 1)$ Axis = x-axis

New parabola

$(x + 2)^2 = \pm 8(y - 1)$

+ve $(x + 2)^2 = 8(y - 1)$

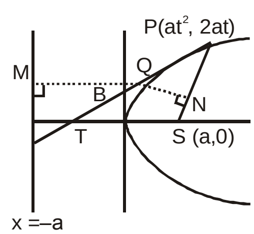
$x^2 + 4x - 8y + 12 = 0$

–ve $x^2 + 4x + 4 + 8y - 8 = 0$

$x^2 + 4x + 8y - 4 = 0$

Sol.3 **A,C**

Tangent at P



$ty = x + at^2$

$B(0, at)$

$T(-at^2, 0)$

clearly

B is the mid point of TP

Sol.4 **A,B**

$y^2 = 4ax$

(A) $(at^2, 2at)$ possible

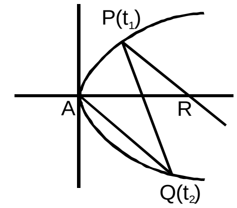
(B) $(at^2, -2at)$ possible

(C) $(a \sin^2 t, 2a \sin t)$ not possible because sin t will lies only in [–1, 1]

so ans. (A) (B)

Sol.5 **C**

Slope of OQ = $\frac{2}{t_2}$



line parallel to AQ and passing through P

$y - 2at_1 = \frac{2}{t_2}(x - at_1^2)$

For point R put y = 0

$-2at_1 = \frac{2}{t_2}(x - at_1^2)$ $t_2 = -t_1 - \frac{2}{t_1}$

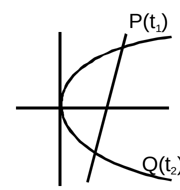
$x = at_1^2 - at_1t_2$ $t_2 + t_1 = -\frac{2}{t_1}$

$= at_1(t_1 - t_2) = 2at_1(t_1 + \frac{1}{t_2})$

$x = 2(at_1^2 + a)$ focal distance

Sol.6 **D**

$T = S_1$



$yy_1 - 2a(x + x_1) = y_1^2 - x_1$

$(x_1, y_1) \Rightarrow (2, 1)$

$y - \frac{2}{4}(x + 2) = 1 - 2$

$4y - 2x = 0$

$x = 2y \Rightarrow$ solve with parabola

$y^2 = 2y$

$y = 0,$ $y = 2$

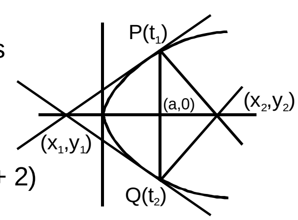
$x = 0,$ $x = 4$

$(0, 0)$ $(4, 2)$

$PQ = \sqrt{4+16} = 2\sqrt{5}$

Sol.7 **C**

POI of Two tangents



$x_1 = at_1t_2$

$y_1 = a(t_1 + t_2)$

POI of Two normals

$x_2 = a(t_1^2 + t_2^2 + t_1t_2 + 2)$

$y_2 = -at_1t_2(t_1 + t_2)$

$y_2 = a(t_1 + t_2)$

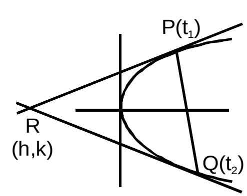
$y_2 = y_1$

Focal chord

$t_1t_2 = -1$

Sol.8 **C**

$h = at_1t_2$



$k = a(t_1 + t_2)$

$k = -\frac{2a}{t_2}$

$t_1 = -\frac{2a}{k}$ $t_2 = -t_1 - \frac{-2}{t_1} \Rightarrow t_2 + t_1 = \frac{-2}{t_1}$

$h = at_1t_2 = at_1\left(-t_1 - \frac{2}{t_1}\right)$

$\Rightarrow h = a\left(-\frac{2a}{k}\right)\left(\frac{2a}{k} + \frac{2}{2a/k}\right) = -\frac{2a^2}{k}\left(\frac{2a}{k} + \frac{k}{a}\right)$

$\Rightarrow\ hk^2 = -4a^3 - 2ak^2 \ \ \Rightarrow\ k^2(h + 2a) + 4a^3 = 0$

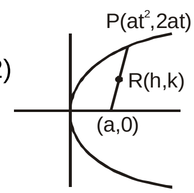
$\Rightarrow\ y^2\,(x + 2a) + 4a^3 = 0$

Sol.9 A,B,C,D

Let R be the mid point

$h = \frac{at^2 + a}{2} \Rightarrow 2h - a = at^2$ ....(1)

$k = \frac{2at + 0}{2} \Rightarrow t = \frac{k}{a}$ ....(2)



$\therefore\ 2h - a = a\left(\frac{k^2}{a^2}\right)$

$\Rightarrow\ k^2 = 2a\left(h - \frac{a}{2}\right) \ \Rightarrow\ y^2 = 2a\left(x - \frac{a}{2}\right)$

vertex $= \left(\frac{a}{2}, 0\right)$ directrix : $x - \frac{a}{2} = -\frac{a}{2}$

Focus $x - \frac{a}{2} = \frac{a}{2}$ $x = 0$ y-axis

$x = a$

Focus (a, 0)

Sol.10 A

Equation of Normal In slope form

$y = mx - 2am - am^3$; $a = \frac{1}{4}$

$6 = 3m - \frac{2m}{4} - \frac{m^3}{4}$ (3, 6)

$m^3 - 10m + 24 = 0 \Rightarrow m = -4$

equation of normal

$y - 6 = -4(x - 3)$

$\Rightarrow\ y + 4x - 18 = 0$

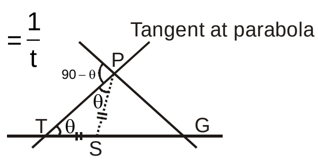
Sol.11 C

Slope of tangent $\tan\theta = t$

$\tan(90 - \theta) = \cot\theta = \frac{1}{t}$

$\tan\theta = t$

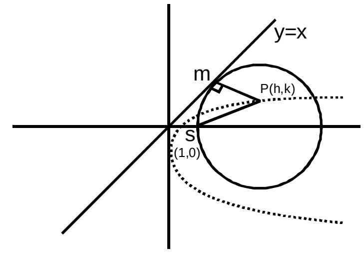
$\theta = \tan^{-1} t$



Sol.12 B,C

$y = \tan(\tan^{-1} x) = x$

$PS = PM \ \Rightarrow\ (h - 1)^2 + (k - 0)^2 = \frac{(h-k)^2}{2}$



$\Rightarrow\ 2(h^2 + 1 - 2h + k^2) = h^2 + k^2 - 2hk$

$\Rightarrow\ h^2 + k^2 + 2hk + 2 - 4h = 0$

$\Rightarrow\ x^2 + y^2 + 2xy + 2 - 4x = 0$

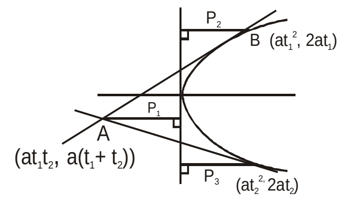
Sol.13 B

$P_2 = at_1^2$

$P_3 = 2t_2^2$

$P_1 = at_1t_2$

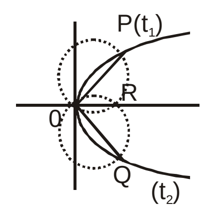
$P_1 = \sqrt{P_2P_3}$



Sol.14 A

$ty = x + at^2$

$\tan\theta_1 = \frac{1}{t_1}$; $\tan\theta_2 = \frac{1}{t_2}$



Circle

$(x - at_1^2)(x - 0) + (y - 0)(y - 2at_1) = 0$

$(x - at_2^2)(x - 0) + (y - 0)(y - 2at_2) = 0$

For Intersection point R

$S_1 - S_2 = 0 \Rightarrow (at_2^2 - at_1^2)x + y(2at_2 - 2at_1) = 0$

$\Rightarrow\ 2y + (t_2 + t_1)x = 0 \ \ \Rightarrow\ y = -\left(\frac{t_1 + t_2}{2}\right)x$

$\tan\theta_1 = \frac{1}{t_1} \ \Rightarrow\ \cot\theta_1 = t_1$ & $\cot\theta_2 = t_2$

$\cot\theta_1 + \cot\theta_2 = t_1 + t_2 = -2\tan\phi$

Sol.15 B

$C_1 : (h - a)^2 + (k - b)^2 = |h|^2$

$C_2 : (h - a)^2 + (k - b)^2 = |k|^2$

$C_1 - C_2 = 0$

$h^2 - k^2 = 0$

$x^2 = y^2$

$y = \pm x$